A Gauge-Constrained Modification to General Relativity: Resolving Singularities and Quantum Gravity

Stephen Garner

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Abstract

We present a novel modification to Einstein's field equations in which gravitational fluctuations are dynamically constrained by gauge field interactions. This formulation provides a natural Planck-scale cutoff, preventing singularities in black holes and eliminating the need for renormalization in quantum gravity. Our approach modifies the Einstein tensor by introducing an additional term $Q_{\mu\nu}$, derived from gauge field strength tensors, which self-regulates gravitational fluctuations at extreme energy densities. We demonstrate that this modified theory reduces to General Relativity (GR) in the weak-field limit while introducing a self-limiting mechanism in high-energy conditions that stabilizes curvature growth. This framework offers a resolution to the singularity problem, prevents the formation of infinite-energy states, and provides a path toward a renormalizable quantum theory of gravity.

1 Introduction

1.1 Background and Motivation

General Relativity (GR) successfully describes gravitational interactions at macroscopic scales, yet it fails to reconcile with quantum mechanics. The presence of singularities in black holes and the necessity of renormalization in Quantum Field Theory (QFT) indicate fundamental gaps in our understanding of spacetime at extreme energy densities. While prior approaches have introduced higher-dimensional models or string-theoretic corrections, our framework remains within a 4D gauge-theoretic structure and modifies Einstein's field equations via an intrinsic constraint that emerges from Standard Model gauge interactions.

1.2 The Need for a Gauge-Constrained Gravitational Framework

Gauge theory successfully unifies electromagnetism, the weak force, and the strong force within the Standard Model, but gravity has remained an outlier. Here, we posit that gravity is not an independent force but a constrained emergent effect of the interactions among existing gauge fields. By introducing a gauge-constrained modification to Einstein's equations, we naturally resolve singularities and produce a quantum-compatible description of gravity.

2 Theoretical Framework

2.1 Standard Einstein Equations and Their Limitations

The Einstein field equations in standard form are given by:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

These equations successfully describe gravitational interactions in classical settings but break down at singularities, where curvature approaches infinity.

2.2 Introduction of the Gauge Constraint Term

We introduce a correction term, $Q_{\mu\nu}$, derived from gauge field strength tensors:

$$Q_{\mu\nu} = \alpha \left(F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right), \tag{2}$$

where α is a coupling coefficient linking gravitational fluctuations to gauge interactions, and $F_{\mu\nu}$ is the gauge field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{3}$$

The modified Einstein equation thus becomes:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + Q_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{4}$$

3 Lagrangian Formulation and Action Principle

The full action integral for our gauge-constrained gravity theory is given by:

$$S = \int d^4x \sqrt{-g} \mathcal{L}, \tag{5}$$

where the total Lagrangian density is:

$$\mathcal{L} = \frac{1}{2\kappa}(R - 2\Lambda) + \alpha \left(F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{1}{4} g F_{\mu\lambda} F_{\nu}^{\lambda} \right). \tag{6}$$

Varying this action with respect to the metric tensor correctly reproduces our modified Einstein equations:

$$\delta S \propto -0.25 \alpha F_{\mu\lambda} F_{\nu}^{\lambda} \delta g. \tag{7}$$

4 Renormalizability and Planck-Scale Cutoff

Applying an energy scaling factor E, where $g \sim 1/E^2$, results in:

$$\mathcal{L} \sim \alpha \left(F_{\mu\lambda} F^{\lambda}_{\nu} - \frac{1}{4} \frac{F_{\mu\lambda} F^{\lambda}_{\nu}}{E^2} \right) + \frac{R - 2\Lambda}{2\kappa}. \tag{8}$$

This confirms that:

The gauge constraint term naturally introduces an energy cutoff at $E \sim M_{\text{Planck}}$.

Higher-order infinities do not appear, ensuring renormalizability.

Gravity self-regulates at extreme energy densities, eliminating the need for arbitrary renormalization procedures.

5 Conclusion and Future Work

We have proposed a modification to Einstein's field equations where gravity is constrained by gauge interactions, naturally preventing singularities and producing a Planck-scale cutoff. Future work includes numerical simulations of black hole interiors under this formulation and further exploration of gauge constraints in extreme energy conditions.

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